

B. TECH
(SEM VI) THEORY EXAMINATION 2022-23
COMPUTER BASED NUMERICAL TECHNIQUES

Time: 3 Hours

Total Marks: 100

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt all questions in brief.

2 x 10 = 20

a. Find the order and degree of

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = k \frac{d^2y}{dx^2}$$

b. Find complementary function of $(D^3 + 1)y = x^3$.

c. Define singular point about a point $x = a$ for the equation $y'' + P_1(x)y' + P_2(x)y = 0$.

d. Prove that $P_n(1) = 1$.

e. Define Gamma function.

f. Write down Bessel's equation.

g. Differentiate a matrix and a determinant.

h. Define orthogonal matrix with an example.

i. How does the choice of boundary conditions influence the solution of unsteady state heat transfer problems?

j. What do you understand by steady state and transient state approaches.

SECTION B

2. Attempt any three of the following:

10x3=30

a. Solve: $\frac{dx}{dt} - \frac{dy}{dt} + 2y = \cos 2t$; $\frac{dx}{dt} + \frac{dy}{dt} + 2x = \sin 2t$.

b. Prove that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$.

c. Show that

$$\sqrt{\frac{1}{2}} \pi x \cdot J_{\frac{3}{2}}(x) = -\sin x - \frac{\cos x}{x}.$$

d.

Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

e. Discuss counter current Liquid-Liquid extraction with an example.

SECTION C

3. Attempt any *one* part of the following: **10x1=10**

- a. Find the complete solution of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x e^{3x} + \sin 2x$.
- b. Solve: $(3y - 2xy^3)dx + (4x - 3x^2y^2)dy = 0$.

4. Attempt any *one* part of the following: **10x1=10**

- a. Solve in series: $x(x-1)y'' + (3x-1)y' + y = 0$.
- b. Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of Legendre polynomials.

5. Attempt any *one* part of the following: **10x1=10**

- a. Prove that $J_0^2 + 2J_1^2 + 2J_2^2 + \dots = 1$.
- b. Evaluate $\int_0^1 x^{n-1} \left[\log_e \left(\frac{1}{x} \right) \right]^{m-1} dx$.

6. Attempt any *one* part of the following: **10x1=10**

- a. Prove that a square matrix is invertible if and only if its determinant is non-zero.
- b. Solve the following equations by matrix method:
 $x + 2y - z = 1$; $3x - 2y + 2z = 2$; $7x - 2y + 3z = 5$.

7. Attempt any *one* part of the following: **10x1=10**

- a. In a refinery of gas stream it is desire to remove 95% of component A from streams containing 10% A. The feed enters in the bottam of a column at a flow rate of 5000kg/hr. The pure solvent is fed at the top of the column at a rate of 5000 kg/hr. Determine the number of trays required by algebraic method, given the equation relation $y = 1.5 x$.
- b. Solve the difference equation $y_{n+2} - 4y_n = n^2 - n - 1$.